

1. Transform the following expressions. Show all work.

a. $\frac{\sin x}{\csc x} + \frac{\cos x}{\sec x}$ to 1

$= \frac{\sin x}{\frac{1}{\sin x}} + \frac{\cos x}{\frac{1}{\cos x}}$ Rewrite denomin.

$= \sin x \cdot \frac{\sin x}{1} + \cos x \cdot \frac{\cos x}{1}$ flip & multiply

$= \sin^2 x + \cos^2 x$ Pyth. Identity

b. $\cos^3 A \sin^2 A = \cos^3 A - \cos^5 A$

$= \cos^3 A (1 - \cos^2 A)$ Subst. using Pyth Identity

$= \cos^3 A - \cos^5 A$ Distribute

2. Prove the following. Show all work.

a. $\frac{\cos^2 \theta}{1 + \sin \theta} = 1 - \sin \theta$

$= \frac{1 - \sin^2 \theta}{1 + \sin \theta}$ Subst. using pyth.

$= \frac{(1 + \sin \theta)(1 - \sin \theta)}{1 + \sin \theta}$ factor numerator

$= 1 - \sin \theta$ Cancel terms

b. $\frac{\sin \theta}{\csc \theta - 1} - \frac{\sin \theta}{\cot^2 \theta} = \tan^2 \theta$

$= \frac{\sin \theta}{\csc \theta - 1} - \frac{\sin \theta}{\csc^2 \theta - 1}$ Subst. using Pyth. Identity

$= \frac{\sin \theta}{(\csc \theta + 1)(\csc \theta - 1)} - \frac{\sin \theta}{(\csc \theta + 1)(\csc \theta - 1)}$ Need common denominator.

$= \frac{(\csc \theta + 1)\sin \theta - \sin \theta}{(\csc \theta + 1)(\csc \theta - 1)}$ combine over 1 fraction

$= \frac{\csc \theta \sin \theta + \sin \theta - \sin \theta}{\csc^2 \theta - 1}$ Distribute

$= \frac{1}{\cot^2 \theta} = \tan^2 \theta$ equals 1, reciprocals

3. Solve the following equations algebraically. Verify your solutions by creating a graph. Show all work.

a. $5 - 7 \sin \theta = 2 \cos^2 \theta, \theta \in [-360^\circ, 450^\circ]$

$5 - 7 \sin \theta = 2(1 - \sin^2 \theta)$

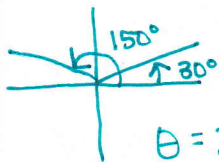
$5 - 7 \sin \theta = 2 - 2 \sin^2 \theta$

$2 \sin^2 \theta - 7 \sin \theta + 3 = 0$

$(2 \sin \theta - 1)(\sin \theta - 3) = 0$

$\sin \theta = \frac{1}{2} \quad \sin \theta = 3$

No Solution



$\theta = 30^\circ + 360^\circ n \quad \& \quad \theta = 150^\circ + 360^\circ n$

$\theta = -330^\circ, -210^\circ, 30^\circ, 150^\circ, 390^\circ$

b. $(\sin x - 3)(\sin x - 1) = 0 \quad x \in [-90^\circ, 270^\circ]$

$\sin x = 3 \quad \sin x = 1$

No Solution

$x = 90^\circ$



c. $\tan^2 x = 2 \tan x \quad x \in [0, 2\pi)$

d. $4 \tan(\theta - 25^\circ) = 7 \quad x \in [-360^\circ, 360^\circ)$

$\tan^2 x - 2 \tan x = 0$

$\tan(\theta - 25^\circ) = \frac{7}{4}$

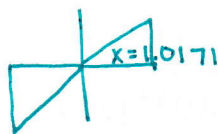
$\tan x (\tan x - 2) = 0$

$\theta - 25^\circ = 60.2551^\circ$

$\tan x = 0 \quad \tan x = 2$

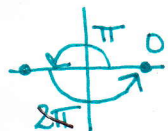
$\theta = 85.2551^\circ + 180^\circ n$

$x = 1.1071 + \pi n$



$\theta = -274.745^\circ, -94.745^\circ, 85.255^\circ$

265.255°

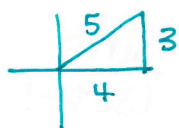


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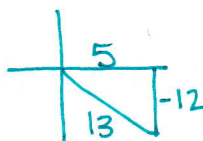
$x = 0, 1.1071, \pi, 4.2487$

4. Calculate the exact values geometrically. Check your answers with your calculator.

a. $\tan(\cos^{-1} \frac{4}{5})$



b. $\cot(\csc^{-1} -\frac{13}{12})$



c. $\cos(\sin^{-1}(-\frac{8}{17}))$



$\tan(\cos^{-1}(\frac{4}{5}))$

$\cot(\csc^{-1}(-\frac{13}{12}))$

$\cos(\sin^{-1}(-\frac{8}{17}))$

$= \frac{3}{4}$

$= \frac{5}{-12}$

$= \frac{15}{17}$

d. $\sin(\sin^{-1} - 1)$

e. $\tan(\cot^{-1} 4)$

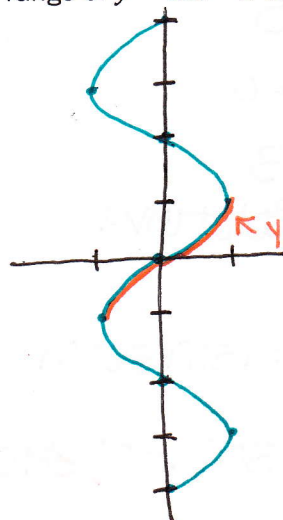
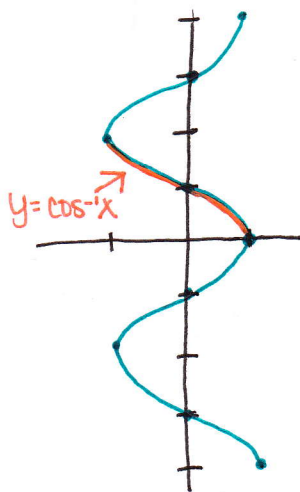
d. $\sin(\sin^{-1} 5)$

$= -1$

$= \frac{1}{4}$

undefined

5. Explain why the range of $y = \cos^{-1} x$ is $[0, \pi]$ but the range of $y = \sin^{-1} x$ is $[-\frac{\pi}{2}, \frac{\pi}{2}]$.



$\sin^{-1} x$ would not pass the VLT from $[0, \pi]$. It would also only cover ratios from 0 to positive 1.